Observations and Challenges

We conclude that

$$Q_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Now try some identities! Multiplying by Q_θ twice should produce $Q_{2\theta}$:

$$Q_{2\theta} = Q_\theta Q_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\text{i.e.} \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

Try also $Q_{\theta+\psi} = Q_\theta Q_\psi$ to get the conventional angle sum identities.

Challenge 1: What matrix R_m represents a reflection in the plane through the line $y = mx$.
Challenge 2: What matrix P_m represents a projection in the plane onto the line $y = mx$.