Linear Mappings Between Vector Spaces

Recall in Topic 2 Part 4 we looked at an $m \times n$ matrix A as a mapping from vectors x in $\mathbb{R}^n$ to vectors y in $\mathbb{R}^m$ through the equation $y = Ax$ (i.e. through the operation of matrix multiplication). We want to continue this now, but with general vector spaces V and W.

Definition: Let V and W be vector spaces. A mapping $T : V \to W$ is a **linear transformation** from V to W if T has the property

$$T(\underbrace{c_1 v_1 + c_2 v_2}_{\text{in } V}) = c_1 \underbrace{T(v_1)}_{\text{in } W} + c_2 \underbrace{T(v_2)}_{\text{in } W}$$

for any scalars c_1, c_2 and any vectors v_1, v_2 in V. (Linear combos map to linear combos with the same coefficients.)(Equivalently $T(u+v) = T(u) + T(v)$, $T(cu) = cT(u)$, i.e. T respects addition and scalar multiplication.)