Left Null Space

Each equation in the restrictions representation of the column space of the edge-node incident matrix is equivalent to an independent small loop in the network.

But what do you do if the small loops are not visually apparent (think of 100,000 nodes!). This is where the **left null space** comes in:

$$A^T y = 0 \Leftrightarrow \begin{array}{rcl} -y_1 - y_2 & = & 0 \\ y_1 - y_3 - y_4 & = & 0 \\ y_2 + y_3 - y_5 & = & 0 \\ y_4 + x_5 & = & 0 \end{array} \overset{\text{G-E}}{\Longleftrightarrow} N(A^T) = \text{Span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

Each basis vector of the left null space of the edge-node incident matrix represents an independent small loop in the network. Positive entries indicate positive flow while negative entries indicate backward flow.