

Column Space

The difference in these two examples lies in “connectedness.” The network represented by A is **connected**, i.e. there is a path (even if you have to go backwards along an edge) from each node to every other node. The graph represented by B is **disconnected**. But we say that it has two **connected components**, nodes 1 and 3, and nodes 2 and 4.

The number of connected components in a graph is equal to the dimension of the null space of the edge-node incident matrix. The non-zero entries in each basis vector indicate the nodes that represent each connected component of the graph.

Look next at the **column space**. We note in example A that Row 1 + Row 3 = Row 2. Also Row 3 + Row 5 = Row 4. For $Ax = b$ this means that

$$b_1 + b_3 = b_2, b_3 + b_5 = b_4 \text{ (conditions restricting } b \text{ for } C(A))$$

Note that we can read these off from the small loops in the graph!