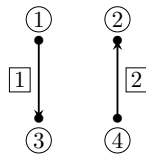


Null Space

For an edge-node incident matrix, each of the four fundamental subspaces captures some information about the structure of the network. Let us begin with the **null space**. For the matrix in our example:

$$Ax = 0 \Leftrightarrow \begin{cases} -x_1 + x_2 = 0 \\ -x_1 + x_3 = 0 \\ -x_2 + x_3 = 0 \\ -x_2 + x_4 = 0 \\ -x_3 + x_4 = 0 \end{cases} \Leftrightarrow \begin{array}{l} \text{all the } x_i\text{'s} \\ \text{are the same} \end{array} \Leftrightarrow N(A) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \right\}$$

Now consider a second example:


$$B = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}, N(B) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$