Examples of Singular Systems

Example with no solutions:

$$\left[\begin{array}{ccc|ccc}
2 & 1 & 1 & 5 \\
0 & -8 & -2 & -12 \\
0 & 0 & 0 & 2
\end{array}\right]$$

 \implies last equation is 0u + 0v + 0w = 2, i.e. 0 = 2 (never satisfied). Note that the last row of the augmented matrix here has a pivot but the coefficient matrix does not!