Proof of the claim $a\mathbb{Z} \subseteq b\mathbb{Z} \iff b|a$

Theorem

$$a\mathbb{Z} \subseteq b\mathbb{Z} \iff b|a \tag{1}$$

Proof.

$\boxed{\Longrightarrow}$

- Assume that $a\mathbb{Z} \subseteq b\mathbb{Z}$.
- In particular this implies $a \in b\mathbb{Z}$.
- Then a is multiple of b.
- Then $b|a$.

$\boxed{\Longleftarrow}$

- Assume $b|a$.
- Then $a = kb$ for some $k \in \mathbb{Z}$.
- Choose $x \in a\mathbb{Z}$, then $x = la$ for some $l \in \mathbb{Z}$.
- Then $x = l(kb) = (lk)b \in b\mathbb{Z}$.

$\square$