Theorem

$$a \equiv b \pmod{n} \iff n|(b-a).$$

Proof, part 2.

$\boxed{\Longleftarrow}$ Let us assume that $n|(b-a)$, or $b-a=kn$. Then if

$$a = q_1 n + r_1,$$
$$b = q_2 n + r_2,$$

we have

$$r_2 - r_1 = b - a + q_1 n - q_2 n = (k + q_1 - q_2)n.$$

This implies that $n|(r_2 - r_1)$. But we also have

$$\left.\begin{array}{l} r_2 < n, \\ r_1 \geq 0 \end{array}\right\} \implies r_2 - r_1 < n, \quad \left.\begin{array}{l} r_1 < n, \\ r_2 \geq 0 \end{array}\right\} \implies r_2 - r_1 > -n$$

and so

$$-n < r_2 - r_1 < n.$$

The only multiple of n that is strictly between $-n$ and n is zero!