Continuity

Definition

We say that $f(x)$ **is continuous at** x_0 if

$$\forall \epsilon > 0, \exists \delta > 0, \forall x \text{ such that } |x - x_0| < \delta, |f(x) - f(x_0)| < \epsilon.$$

To negate this, we also use the fact that

$$\neg(P \implies Q) \iff \neg(\neg P \vee Q) \iff P \wedge \neg Q.$$

$$\begin{aligned}
&\neg(\forall \epsilon > 0, \exists \delta > 0, \forall x, |x - x_0| < \delta \implies |f(x) - f(x_0)| < \epsilon) \iff \\
&\exists \epsilon > 0, \neg(\exists \delta > 0, \forall x, |x - x_0| < \delta \implies |f(x) - f(x_0)| < \epsilon) \iff \\
&\exists \epsilon > 0, \forall \delta > 0, \neg(\forall x, |x - x_0| < \delta \implies |f(x) - f(x_0)| < \epsilon) \iff \\
&\exists \epsilon > 0, \forall \delta > 0, \exists x \neg(|x - x_0| < \delta \implies |f(x) - f(x_0)| < \epsilon) \iff \\
&\exists \epsilon > 0, \forall \delta > 0, \exists x(|x - x_0| < \delta \wedge |f(x) - f(x_0)| \geq \epsilon)
\end{aligned}$$