

## Theorem

$\sqrt{2}$  is irrational.

When we have been given almost nothing, let's go with contradiction, i.e.  
**Assume it's false and let 'er rip**

## Proof.

Let us assume that  $\sqrt{2}$  is rational, then  $\sqrt{2} = \frac{p}{q}$  for  $p, q \in \mathbb{Z}$ .

Assume that  $p, q$  have no common divisors (the fraction is in “reduced form”).

By algebra, this gives  $2q^2 = p^2$ .

From this, we have  $p^2$  is even (it is  $2 * \text{something}$ ).

From Corollary, this means  $p$  is even, or  $p = 2k$  for some  $k \in \mathbb{Z}$ .

Plug back in, we get  $2q^2 = 4k^2$  or  $q^2 = 2k^2$ .

Therefore  $q^2$  is even and thus  $q$  is even.

Both  $p$  and  $q$  are even, but they cannot have a common divisor!

**CONTRADICTION**

