Parity

Theorem

Let $m, n \in \mathbb{Z}$. Then m and n are both odd if and only if $(m \times n)$ is odd.

Corollary

n and n^2 have the same parity, i.e.

1. *n is odd $\iff$ n^2 is odd;*
2. *n is even $\iff$ n^2 is even.*

Proof.

Note that (1) follows directly from the theorem by choosing $m = n$.
For (2), note that $\neg P \iff \neg Q$ is the same as $P \iff Q$! □