Example

Theorem

Let $m, n \in \mathbb{Z}$. Then m and n are both odd if and only if $(m \times n)$ is odd.

Proof.

We will do both directions.
$\boxed{\Rightarrow}$ Assume that m, n are both odd.
Then $m = 2k + 1$ and $n = 2l + 1$ for $k, l \in \mathbb{Z}$.
Then we have

$$mn = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1,$$

which is odd.
$\boxed{\Leftarrow}$ We will use $\neg P \implies \neg Q$, which is equivalent to $Q \implies P$.
If it is false that m and n are both odd, then at least one of them must be even.
Without loss of generality, assume $m = 2k$. Then $mn = 2kn = 2(kn)$ is even.

□

Why not $Q \implies P$ here?

Let's try it. Assume mn is odd. Then $mn = 2k + 1$ for some $k \in \mathbb{Z}$....
...and then I dunno(?)