Theorem

If $x + 3$ is even, then x is odd.

- **Direct.** Assume $x + 3$ is even.
  This means that $x + 3 = 2k$ for some $k \in \mathbb{Z}$.
  Then $x = 2k - 3 = 2(k - 2) + 1$ and therefore x is odd.
- **Contrapositive.** Assume that x is even.
  Then $x = 2k$ for some $k \in \mathbb{Z}$.
  Then $x + 3 = 2k + 3 = 2(k + 1) + 1$ is odd.
- **Contradiction.** Assume that $x + 3$ is even and x is even.
  This means that $x + 3 = 2k$ and $x = 2l$ for $k, l \in \mathbb{Z}$, or $2l = 2k + 3$.
  This means that $l - k = 3/2$, which is not possible if $k, l \in \mathbb{Z}$.