Theorem

Let $k, l \in \mathbb{Z}$. If $k + l$ is odd, then one of k, l is even and the other is odd.

Note that we have

- $P =$ "$k + l$ is odd", and
- $Q =$ "one of k, l is even and the other is odd.
- Note that Q is more specific than P.

Proof.

- Assume $\neg Q$, so that k, l have the same parity.
- (Exhaustion!)
- If k, l even then $k = 2a, l = 2b$ and $k + l = 2a + 2b = 2(a + b)$ is even.
- If k, l off then $k = 2a + 1, l = 2b + 1$, and
  $k + l = 2a + 1 + 2b + 1 = 2a + 2b + 2 = 2(a + b + 1)$ is even.
- Boom, roasted

□

Both strategy and tactics

Note here that the strategy was contrapositive, but the tactic was Cases/Exhaustion