- Since $E[Y_{n+1}] = \mathbb{E}[Y_n]$,
- by induction we have
$$\mathbb{E}[Y_n] = \mathbb{E}[Y_0] = 0.$$
- Let
  - $A(n)$ be the probability that Alice has won by round n;
  - $B(n)$ be the probability that Bob has won by round n;
  - $C(n)$ be the probability that neither player has yet won.
- Clearly we have
$$A(n) + B(n) + C(n) = 1,$$

Law of Total Expectation

- We have

$$\begin{aligned} 0 &= \mathbb{E}[Y_n] \\ &= \mathbb{E}[Y_n | Y_n = y]A(n) + \mathbb{E}[Y_n | Y_n = -x]B(n) \\ &\quad + \mathbb{E}[Y_n | -x < Y_n < y]C(n) \\ &= yA(n) - xB(n) + \mathbb{E}[Y_n | -x < Y_n < y]C(n). \end{aligned}$$