- We will let $Y_n$ denote the amount of money that Alice has won after $n$ rounds;
- (We define $Y_0 = 0$)
- Then:
  - If $Y_n = y$, then $Y_{n+1} = y$ (Alice already won everything);
  - If $Y_n = -x$, then $Y_{n+1} = -x$ (Bob already won everything);
  - If $-x < Y_n < y$, then

$$Y_{n+1} = \begin{cases} Y_n + 1, & \text{probability } 1/2, \\ Y_n - 1, & \text{probability } 1/2. \end{cases}$$

Note that $\mathbb{E}[Y_{n+1}] = \mathbb{E}[Y_n]$:

$$\begin{aligned} \mathbb{E}[Y_{n+1}] &= \mathbb{E}[Y_{n+1}|H]\mathbb{P}(H) + \mathbb{E}[Y_{n+1}|T]\mathbb{P}(T) \\ &= \mathbb{E}[Y_n + 1] \cdot \frac{1}{2} + \mathbb{E}[Y_n - 1] \cdot \frac{1}{2} = \mathbb{E}[Y_n]. \end{aligned}$$