Final answer

If

$$\hat{A} + \hat{B} = 1,$$
$$y\hat{A} - x\hat{B} = 0,$$

then

$$\hat{A} = \frac{x}{x+y}, \qquad \hat{B} = \frac{y}{x+y}.$$

Paradox

Even though the game is fair — the final result is not!