Theorem

Let X, Y be independent random variables. Then

$$G_{X+Y}(t) = G_X(t) \cdot G_Y(t).$$

More generally, if $X_1, \ldots, X_n$ are independent, then

$$G_{(\sum_i X_i)}(t) = \prod_i G_{X_i}(t).$$

Proof

- The second expression follows from the first by induction, so ...
- We have

$$G_{X+Y}(t) = \mathbb{E}[t^{X+Y}] = \mathbb{E}[t^X t^Y].$$

Recall that X, Y are independent, so

$$\mathbb{E}[t^X t^Y] = \mathbb{E}[t^X]\mathbb{E}[t^Y] = G_X(t)G_Y(t).$$