

Theorem

Let X, Y be independent random variables. Then

$$G_{X+Y}(t) = G_X(t) \cdot G_Y(t).$$

More generally, if X_1, \dots, X_n are independent, then

$$G_{(\sum_i X_i)}(t) = \prod_i G_{X_i}(t).$$

Basically...

Adding **independent** random variables is the same as multiplying their PGFs...