More properties

- $G_X(1) = 1$ for any X;
- $\frac{dG_X}{dt}(1) = \mathbb{E}[X]$.

proof

- Since $1^X = 1$, then $\mathbb{E}[1^X] = \mathbb{E}[1] = 1$.
- If we differentiate

$$\mathbb{E}[t^X] = \sum_{k=0}^{\infty} t^k \mathbb{P}(X = k),$$

we get

$$\frac{d}{dt}\mathbb{E}[t^X] = \sum_{k=0}^{\infty} kt^{k-1}\mathbb{P}(X = k)$$

and plugging in $t = 1$ gives

$$\sum_{k=0}^{\infty} k\mathbb{P}(X = k) = \mathbb{E}[X].$$