

Bernoulli trials

- Do n trials, independent, success with probability p .
- Number of successes is $\text{Bin}(n, p)$, and

$$\mathbb{P}(\text{Bin}(n, p) = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

Let's try something

- Let $f(t) = pt + (1-p)$.
- Then

$$\begin{aligned} (f(t))^n &= (pt + (1-p))^n = \sum_{k=0}^n \binom{n}{k} (pt)^k (1-p)^{n-k} \\ &= \sum_{k=0}^n \left[\binom{n}{k} p^k (1-p)^{n-k} \right] t^k \\ &= \sum_{k=0}^n [\mathbb{P}(\text{Bin}(n, p) = k)] t^k. \end{aligned}$$