More generally

- We can compute the nth derivative

$$f^{(n)}(t) = (3 \cdot 4 \cdot 5 \cdot (n+2))(2-t)^{-(n+3)} = \frac{(n+2)!}{2}(2-t)^{-(n+3)}.$$

- and thus

$$\frac{f^{(n)}(0)}{n!} = \frac{(n+2)!}{2 \cdot n!} 2^{-(n+3)}.$$

- So

$$f(t) = \sum_{n=0}^{\infty} \frac{(n+2)!}{2 \cdot n!} 2^{-(n+3)} t^n.$$

Then

- Recall $G_X(t) = t^3 f(t)$, so

$$\mathbb{P}(X = k) = \frac{(k-1)!}{2(k-3)!} 2^{-k}, \quad k \geq 3.$$