

Recall "flip until first head"

- Say we flip coins until we get first head, then count number of flips;
- We computed:

$$\mathbb{P}(F = n) = 2^{-n}.$$

- So

$$G_F(t) = \sum_{k=1}^{\infty} 2^{-k} t^k = \sum_{k=1}^{\infty} \left(\frac{t}{2}\right)^k = \frac{t/2}{1 - t/2} = \frac{t}{2 - t}.$$

Ok, how about if we flip until three heads?

- This means playing the game three times, and restarting;
- So we have F_1, F_2, F_3 with the same distribution as above, and

$$X = F_1 + F_2 + F_3.$$

- Therefore

$$G_X(t) = \left(\frac{t}{2 - t}\right)^3.$$