

Statistics angle on this

- Let's say we have a "black box" experiment with probability p ,
- but p is unknown...
- One thing to try: do n trials of the experiment, define

$$\hat{p} := \frac{\# \text{ successes}}{n}.$$

- So maybe guess \hat{p} as a "best guess" for p . Is it a good guess?

- We know

$$\mathbb{P}(|\hat{p} - p| \geq \epsilon) \leq \frac{pq}{n\epsilon^2} \leq \frac{1}{4n\epsilon^2}.$$

- So let's try "95% confidence interval": choose $\tilde{\epsilon}$ so RHS = 5%:

$$\frac{1}{4n\tilde{\epsilon}^2} = \frac{1}{20} \implies \tilde{\epsilon} = \sqrt{\frac{1}{5n}},$$

and we say: "We are 95% confident that

$$\hat{p} - \tilde{\epsilon} \leq p \leq \hat{p} + \tilde{\epsilon}."$$