Proof of LLN

Since

$$\mathbb{E}[A_n] = \mu, \qquad \mathrm{Var}(A_n) = \frac{\mathrm{Var}(X)}{n},$$

we have

$$\mathbb{P}(|A_n - \mu| \geq \epsilon) \leq \frac{\mathrm{Var}(A_n)}{\epsilon^2} \leq \frac{\mathrm{Var}(X)}{n\epsilon^2}$$

and this goes to zero as $n \to \infty$.

Note

- For any $\epsilon > 0$, this goes to zero ...
- but if ϵ is small, then it "takes longer" to go to zero.