Theorem (Law of Large Numbers)

Let X_i be IID, $\mu = \mathbb{E}[X_i]$. Assume that μ and $\mathrm{Var}(X_i)$ are both finite. Then for any $\epsilon > 0$,

$$\lim_{n \to \infty} \mathbb{P}(|A_n - \mu| < \epsilon) = 1.$$

Note what we do *not* say:

- We don't say $A_n = \mu$
- We don't even say that, for any fixed n,

$$\mathbb{P}(|A_n - \mu| < \epsilon) = 1.$$

- (It only works in the limit)

Also!

Note that we pick ϵ, then let $n \to \infty$.