

## Theorem

Let  $X_i$  be IID, let  $\mu = \mathbb{E}[X_1]$ . Define

$$S_n = X_1 + X_2 + \cdots + X_n = \sum_{i=1}^n X_i, \quad A_n = S_n/n.$$

Then

$$\mathbb{E}[A_n] = \mu, \quad \text{Var}(A_n) = \frac{\text{Var}(X_1)}{n}.$$

- $A_n$  has mean  $\mu$ , and;
- $A_n$  has small variance when  $n$  large;
- So (in some sense) “ $A_n$  is close to  $\mu$ ” when  $n$  large