Example

- Flip 10 coins, fair, independent. Let X be number of heads. What is $V(X)$?
- Let $X_i = 1$ if the ith coin is heads, 0 if tails, and $X = \sum_{i=1}^{10} X_i$.
- We know that $\mathbb{E}[X_i] = 1/2$ and thus $\mathbb{E}[X] = 10 \cdot 1/2 = 5$.
- We can compute $V(X_i)$ two ways:
  - Note that $X_i^2 = X_i$, so $\mathbb{E}[X_i^2] = \mathbb{E}[X_i] = 1/2$ and thus
$$V(X_i) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$
  - We can also compute
$$V(X_i) = \mathbb{E}[(X_i - 1/2)^2] = \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{4}.$$
- Therefore, since X_i are independent, we have
$$V(X) = 10V(X_1) = \frac{10}{4} = \frac{5}{2}.$$