Theorem

Let X, Y be **independent** *random variables. Then*

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y).$$

Proof of theorem

- Let us write $\mathbb{E}[X] = a$, $\mathbb{E}[Y] = b$. Then $\mathbb{E}[X+Y] = a+b$.
- Then we have

$$\begin{aligned}
\mathrm{Var}(X+Y) &= \mathbb{E}[(X+Y)^2] - (\mathbb{E}[X+Y])^2 \\
&= \mathbb{E}[X^2 + 2XY + Y^2] - (a+b)^2 \\
&= \mathbb{E}[X^2] + 2\mathbb{E}[XY] + \mathbb{E}[Y^2] - (a^2 + 2ab + b^2) \\
&= (\mathbb{E}[X^2] - a^2) + 2(\mathbb{E}[XY] - ab) + (\mathbb{E}[Y^2] - b^2) \\
&= (\mathbb{E}[X^2] - a^2) + 2(\mathbb{E}[X]\mathbb{E}[Y] - ab) + (\mathbb{E}[Y^2] - b^2) \\
&= \mathrm{Var}(X) + 0 + \mathrm{Var}(Y).
\end{aligned}$$