

## Theorem

Let  $X, Y$  be **independent** random variables. Then

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y).$$

## Lemma!

If  $X, Y$  are **independent** random variables, then

$$\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y].$$

## Proof

$$\begin{aligned}\mathbb{E}[XY] &= \sum_n n \cdot \mathbb{P}(XY = n) = \sum_{k,l} kl \cdot \mathbb{P}(X = k \wedge Y = l) \\ &= \sum_{k,l} kl \cdot \mathbb{P}(X = k)\mathbb{P}(Y = l) \\ &= \sum_k k \cdot \mathbb{P}(X = k) \sum_l l \cdot \mathbb{P}(Y = l) \\ &= \mathbb{E}[X]\mathbb{E}[Y].\end{aligned}$$