Some properties of variance

- $\text{Var}(\alpha X) = \alpha^2 \, \text{Var}(X)$
- $\text{Var}(X + c) = \text{Var}(X)$

Proof of (1)

- Let us write $Y = \alpha X$. We have

$$\mathbb{E}[Y] = \mathbb{E}[\alpha X] = \alpha \mathbb{E}[X].$$

- Write $\mu_X = \mathbb{E}[X]$ and $\mu_Y = \mathbb{E}[Y]$, and $\mu_Y = \alpha \mu_X$.
- Then

$$\begin{aligned}
\text{Var}(Y) &= \mathbb{E}[(Y - \mu_Y)^2] = \mathbb{E}[(\alpha X - \alpha \mu_X)^2] \\
&= \mathbb{E}[\alpha^2 (X - \mu_X)^2] = \alpha^2 \mathbb{E}[(X - \mu_X)^2] = \alpha^2 \, \text{Var}(X).
\end{aligned}$$