

Theorem (Really Useful Result!)

$$V(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2.$$

Proof

- We compute:

$$\begin{aligned} V(X) &= \mathbb{E}[(X - \mu)^2] \\ &= \mathbb{E}[X^2 - 2\mu X + \mu^2] \\ &= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2 \\ &= \mathbb{E}[X^2] - 2\mu^2 + \mu^2 \\ &= \mathbb{E}[X^2] - \mu^2. \end{aligned}$$