Definition of IID

We say that a sequence $X_1, X_2, \ldots, X_n$ of random variables is **independent and identically distributed (IID)** if

1. They are independent;
2. All X_i have the same distribution.

Theorem

Let X_i be IID. Define

$$S_n = X_1 + X_2 + \cdots + X_n = \sum_{i=1}^{n} X_i,$$

$$A_n = S_n/n.$$

Then

$$\mathbb{E}[S_n] = n \cdot \mathbb{E}[X_1], \qquad \mathrm{Var}(S_n) = n\,\mathrm{Var}(X_1),$$

$$\mathbb{E}[A_n] = \mathbb{E}[X_1], \qquad \mathrm{Var}(A_n) = \frac{\mathrm{Var}(X_1)}{n}.$$