Variance

- Let $X : \Omega \to \mathbb{R}$ be a random variable.
- The **expected value**, or **expectation** of X is the sum

$$\mathbb{E}[X] := \sum_{\omega \in \Omega} X(\omega) p(\omega)$$

- Say $\mathbb{E}[X] = \mu$. Then the **variance** of X, denote $\mathrm{Var}(X)$ or $V(X)$, is

$$\begin{aligned} \mathrm{Var}(X) &= \mathbb{E}[(X - \mu)^2] \\ &= \sum_k (k - \mu)^2 \mathbb{P}(X = k). \end{aligned}$$

- We also define the **standard deviation** of X by

$$\sigma(X) = \sqrt{\mathrm{Var}(X)}.$$

Note

We need to compute $\mathbb{E}[X]$ first, then compute $V(X)$.