Theorem

Let X, Y be random variables, and $\alpha, \beta \in \mathbb{R}$. Then

$$\mathbb{E}[\alpha X + \beta Y] = \alpha \mathbb{E}[X] + \beta \mathbb{E}[Y].$$

We have

$$\begin{aligned}
\mathbb{E}[\alpha X + \beta Y] &= \sum_{\omega \in \Omega} (\alpha X + \beta Y)(\omega) p(\omega) \\
&= \sum_{\omega \in \Omega} (\alpha X(\omega) + \beta Y(\omega)) p(\omega) \\
&= \alpha \sum_{\omega \in \Omega} X(\omega) p(\omega) + \beta \sum_{\omega \in \Omega} Y(\omega) p(\omega) \\
&= \alpha \mathbb{E}[X] + \beta \mathbb{E}[Y].
\end{aligned}$$