Theorem

An alternative method to compute expectation is

$$\mathbb{E}[X] = \sum_k k \cdot \mathbb{P}(X = k).$$

Proof

- By definition, we have

$$\begin{aligned}
\sum_k k \cdot \mathbb{P}(X = k) &= \sum_k k \sum_{\omega : X(\omega) = k} p(\omega) \\
&= \sum_k \sum_{\omega : X(\omega) = k} k p(\omega) \\
&= \sum_k \sum_{\omega : X(\omega) = k} X(\omega) p(\omega) \\
&= \sum_{\omega \in \Omega} X(\omega) p(\omega).
\end{aligned}$$