Law of Total Expectation

Let $A_1, A_2, \ldots, A_n$ be a partition of Ω. Then

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[X|A_i]\mathbb{P}(A_i).$$

Proof

We compute:

$$\begin{aligned}
\sum_{i=1}^{n} \mathbb{E}[X|A_i]\mathbb{P}(A_i) &= \sum_{i=1}^{n} \sum_{k} k\mathbb{P}(X = k|A_i)\mathbb{P}(A_i) \\
&= \sum_{k} k \sum_{i=1}^{n} \mathbb{P}(X = k|A_i)\mathbb{P}(A_i) \\
&= \sum_{k} k\mathbb{P}(X = k) \qquad \text{(Law of Total Probability)} \\
&= \mathbb{E}[X].
\end{aligned}$$