

Thus

$$\mathbb{P}(\text{exactly } k \text{ heads in } 2k \text{ flips}) \approx \frac{1}{\sqrt{\pi k}}.$$

Back to the original question

$$\frac{1}{\sqrt{500\pi}} = 0.02523132522020 \dots$$
$$\binom{1000}{500} 2^{-1000} = 0.0252250181783 \dots$$