

Stirling's Formula

Here is a useful formula:

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}.$$

The probability of exactly half heads

Let $n = 2k$, probability of k heads.

$$\begin{aligned} \binom{2k}{k} 2^{-2k} &= \frac{(2k)!}{(k!)^2} 4^{-k} \\ &\approx 4^{-k} \left(\frac{2k}{e}\right)^{2k} \sqrt{2\pi(2k)} / \left(\left(\frac{k}{e}\right)^k \sqrt{2\pi k} \right)^2 \\ &= 4^{-k} \frac{4^k k^{2k} e^{-2k} \cdot 2\sqrt{\pi k}}{k^{2k} e^{-2k} (2\pi k)} \\ &= \frac{1}{\sqrt{\pi k}}. \end{aligned}$$