Definition

We say that A, B are **independent** if $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$.

Note

- Assume that $\mathbb{P}(A), \mathbb{P}(B) > 0$, and A, B are independent.
- Then
$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(A)\mathbb{P}(B)}{\mathbb{P}(B)} = \mathbb{P}(A).$$
- and
$$\mathbb{P}(B|A) = \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)} = \frac{\mathbb{P}(B)\mathbb{P}(A)}{\mathbb{P}(A)} = \mathbb{P}(B).$$

Definition

We say that the random variables X, Y are **independent** if $\forall k, l$:

$$\mathbb{P}(X = k \wedge Y = l) = \mathbb{P}(X = k) \cdot \mathbb{P}(Y = l).$$