

Roll two dice

- Let X_1, X_2 be the results of rolling two dice. Assume that each outcome is equally likely, so $p(i, j) = 1/36$ for all $i, j = 1, \dots, 6$.
- Let $Y = X_1 + X_2$.

$$\begin{aligned}\mathbb{P}(X_1 = 1 | Y = 3) &= \frac{\mathbb{P}(X_1 = 1 \wedge Y = 3)}{\mathbb{P}(Y = 3)} \\ &= \frac{\mathbb{P}(X_1 = 1 \wedge X_2 = 2)}{\mathbb{P}(Y = 3)} = \frac{1/36}{2/36} = \frac{1}{2}.\end{aligned}$$

$$\begin{aligned}\mathbb{P}(Y = 3 | X_1 = 1) &= \frac{\mathbb{P}(X_1 = 1 \wedge Y = 3)}{\mathbb{P}(X_1 = 1)} \\ &= \frac{\mathbb{P}(X_1 = 1 \wedge X_2 = 2)}{\mathbb{P}(X_1 = 1)} = \frac{1/36}{1/6} = \frac{1}{6}.\end{aligned}$$