Theorem

If A_n form a partition of Ω, then for any event E,

$$\mathbb{P}(E) = \sum_{i=1}^{n} \mathbb{P}(E \cap A_i).$$

Proof

Let $E_i = E \cap A_i$. Note that if $i \neq j$,

$$E_i \cap E_j = (E \cap A_i) \cap (E \cap A_j) = E \cap A_i \cap A_j = \emptyset.$$

We also have

$$\bigcup_{i=1}^{n} E_i = \bigcup_{i=1}^{n} (E \cap A_i) = E \cap \bigcup_{i=1}^{n} A_i = E \cap \Omega = E.$$

Then

$$\mathbb{P}(E) = \mathbb{P}\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} \mathbb{P}(E_i).$$