Proof.

3. We have

$$\mathbb{P}(B) = \sum_{\omega \in B} p(\omega) = \sum_{\omega \in A} p(\omega) + \sum_{\omega \in (B \setminus A)} p(\omega),$$
$$= \mathbb{P}(A) + \sum_{\omega \in (B \setminus A)} p(\omega).$$

Since the last sum is ≥ 0, we have $\mathbb{P}(B) \geq \mathbb{P}(A)$.

4. Since $A \cap B = \emptyset$, if $x \in A \cup B$, then $x \in A$ or $x \in B$ **but not both**. Then

$$\mathbb{P}(A \cup B) = \sum_{\omega \in (A \cup B)} p(\omega) = \sum_{\omega \in A} p(\omega) + \sum_{\omega \in B} p(\omega)$$
$$= \mathbb{P}(A) + \mathbb{P}(B).$$

5. Note $A \cup A^c = \Omega$ and $A \cap A^c = \emptyset$. Then

$$1 = \mathbb{P}(\Omega) = \mathbb{P}(A) + \mathbb{P}(A^c),$$

and solve for $\mathbb{P}(A^c)$.

□