Theorem

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Let Ω be discrete probability space, and let A, B be events. Then:

1. $\mathbb{P}(A) \geq 0$;
2. $\mathbb{P}(\Omega) = 1$;
3. $A \subseteq B \implies \mathbb{P}(A) \leq \mathbb{P}(B)$;
4. *If* $A \cap B = \emptyset$, *then*
$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B).$$
5. $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$.

Proof.

1. We have
$$\mathbb{P}(A) = \sum_{\omega \in A} p(\omega) \geq 0. \tag{1}$$
2. We have
$$\mathbb{P}(\Omega) = \sum_{\omega \in \Omega} p(\omega),$$
and this sum is 1 by definition.

□