Example of nonconstructive proof

Theorem

There are a, b irrational such that a^b is rational.

Proof.

Let us consider the three numbers

$$\sqrt{2}, \quad \sqrt{2}^{\sqrt{2}}, \quad (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}.$$

First note, from the rules of exponents, that

$$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = (\sqrt{2})^{\sqrt{2}\cdot\sqrt{2}} = (\sqrt{2})^2 = 2,$$

and that is surely rational!

So if $\sqrt{2}^{\sqrt{2}}$ is irrational, then we are done: $a = \sqrt{2}^{\sqrt{2}}$, $b = \sqrt{2}$.

But what if $\sqrt{2}^{\sqrt{2}}$ is rational? Well then choose $a = b = \sqrt{2}$.

$\square$

The proof didn't tell us whether or not $\sqrt{2}^{\sqrt{2}}$ is rational!