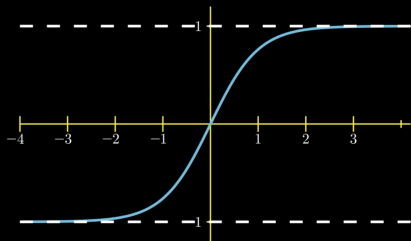


**Exercise:** Show that  $|(-1, 1)| = |\mathbb{R}|$ .

That is to say: the cardinality of this open interval is the same as the real line.



$$y = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

continuous and increasing

$$\lim_{x \rightarrow \infty} \tanh(x) = 1$$

$$\lim_{x \rightarrow -\infty} \tanh(x) = -1$$

increasing  $\Rightarrow$  injective,    continuous  $\Rightarrow$  surjective

Therefore  $\tanh$  is a bijection from  $\mathbb{R}$  to  $(-1, 1)$ .