Theorem

For any $a < b$, and any $c < d$, the intervals $(a, b), [a, b], (c, d), [c, d]$ all have the same cardinality.

- We can check that the linear map

$$f(x) = \frac{d - c}{b - a} x + \frac{bc - ad}{b - a}$$

  maps $a \rightarrow c$ and $b \rightarrow d$.
- Since the coefficient of the linear term is nonzero, it is a bijection.
- This shows that $|[a, b]| = |[c, d]|$
- Use previous slide's idea to show that $|[a, b]| = |(a, b)|$, etc.