A truly wacky set

- Pick $\epsilon > 0$.
- Order the rational numbers however you like:

$$q_1, q_2, q_3, q_4, \ldots, q_n, \ldots$$

- Let A_n be the open interval centered at q_n with width $\epsilon 2^{-n}$:

$$A_n = \left(q_n - \frac{\epsilon}{2^{n+1}}, q_n + \frac{\epsilon}{2^{n+1}}\right).$$

- Let

$$A = \bigcup_{n=1}^{\infty} A_n.$$

- Since the nth interval has "width" $\epsilon 2^{-n}$, the "total mass" of the set A is $\leq$

$$\sum_{n=1}^{\infty} \frac{\epsilon}{2^n} = \epsilon \sum_{n=1}^{\infty} \frac{1}{2^n} = \epsilon.$$

- But it has cardinality equal to $\mathbb{R}$...