

## Theorem

If  $A_n$  is countable for every  $n$ , then  $\bigcup_{n \in \mathbb{N}} A_n$  is also countable.

## Proof.

Let us write

$$A_n = (a_{n1}, a_{n2}, \dots, a_{nk}, \dots). \quad (1)$$

Then reorder the terms like so:

$$(a_{11}, a_{12}, a_{21}, a_{13}, a_{22}, a_{31}, a_{14}, a_{23}, a_{32}, a_{41}, \dots)$$



<del><math>a_{11}</math></del>	<del><math>a_{12}</math></del>	<del><math>a_{13}</math></del>	<del><math>a_{14}</math></del>	$\dots$
<del><math>a_{21}</math></del>	$a_{22}$	<del><math>a_{23}</math></del>	<del><math>a_{24}</math></del>	$\dots$
<del><math>a_{31}</math></del>	<del><math>a_{32}</math></del>	$a_{33}$	<del><math>a_{34}</math></del>	$\dots$
<del><math>a_{41}</math></del>	$a_{42}$	$a_{43}$	<del><math>a_{44}</math></del>	$\dots$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$