Theorem

If A_n is countable for every n, then $\cup_{n\in\mathbb{N}} A_n$ is also countable.

Proof.

Let us write

$$A_n = (a_{n1}, a_{n2}, \dots, a_{nk}, \dots). \tag{1}$$

Then reorder the terms like so:

$$(a_{11}, a_{12}, a_{21}, a_{13}, a_{22}, a_{31}, a_{14}, a_{23}, a_{32}, a_{41}, \dots)$$

□