Big result

Theorem

$\mathbb{Q}$ is countable.

Note!

This implies that we can list all of the rational numbers!

Theorem

If A_n is countable for every n, then $\cup_{n \in \mathbb{N}} A_n$ is also countable.

Proof that second theorem gives first.

For each n, define
$$A_n = \{k/n : k \in \mathbb{Z}\}.$$

Note that $\mathbb{Q} = \cup_{n \in \mathbb{N}} A_n$. $\qquad\square$